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Analysis of Students' Errors in Solving Function Derivative Application Problems Based on Watson's Error Criteria

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***Abstract*— In adressing problems related to the application of derivatives of function, many students demonstrate inadequate mastery of derivative concepts. It leads to various mistakes in solving derivative application problems. This study aims to analyze student's errors in solving derivative application problems based on Watson's error criteria. This research employs a qualitative method with descriptive approach. The data analyzed in this study were derived from the formative test on the topic of derivatives of functions administered to students in Class XII MIPA 3. Three students were selected as subjects based on the types of errors they have made. The instruments were developed based on tests results and interviews, which were then analyzed using the Watson's error criteria. The interviews employed semi-structured protocol to gain deeper insights about the errors made by the students in solving derivative application problems. The interview results were presented in the form of transcripts. Results showed that the types of student's errors are include incorrect procedural steps, missing data, incomplete conclusions, hierarchy skill issues, and response-level conflicts. These findings are expected to serve as a reference for future research and to provide insights for mathematics teachers of the potential errors made by students, particularly in solving derivative application problems.**

Keywords— ***Error Analysis; Problem Solving; Function Derivative Applications***

I. INTRODUCTION

Mathematics is an essential subject that must be learned by all students at various education levels. During Mathematics lessons, a high level of understanding is required. According to Haryono et al [1], mathematics is one of the essential subjects that students need in solving real-life problems. In learning mathematics, students are not only expected to understand mathematical concepts but also to apply those concepts in daily life problems.

In mathematics learning, students are expected to develop critical, logical, precise, effective, and efficient thinking skills in problem-solving. It is a complex process requiring the coordination of specific or general knowledge possessed by each individual. According to Siswanto and Meiliasari [2], problem-solving is a fundamental aspect of the mathematics curriculum, as it provides students with opportunities to apply their knowledge and skills in addressing complex problems. Both in the learning process and its practical application, problem-solving fosters critical thinking and the ability to navigate mathematical challenges effectively. Through problem-solving in mathematics, students are expected to develop systematic thinking, guiding them in investigating solutions and gaining meaningful learning experiences. This process enhances critical and analytical thinking, which is highly beneficial in preparing them to face global challenges.

Mathematical problems are interpreted differently by each individual (Laamena et al., 2021) [3]. Some students can solve problems with correct steps, others can only partially solve them, some make minor errors, while others commit significant errors during the process, resulting incorrect answers.

According to Fadaukas and Khasanah [4], An analysis of students' errors in solving trigonometric function problems is essential as an evaluation tool for educators in designing appropriate teaching strategies and methods that align with the conditions and characteristics of each class. Through this analysis, similar errors can be minimized, thereby enhancing students' conceptual understanding and improving their mathematics achievement in school. To identify the errors made by students, an error analysis should be performed using the error criteria outlined by Watson (Suryani et al. [5]). Watson's criteria are used to analyze students' mistakes. The categories of errors according to Watson are suitable for analyzing errors made by students in Mathematics and Natural Science (MIPA) subjects.

One of the mathematical concepts that is frequently used in daily life is the derivative of a function. The topic of derivatives is crucial to form the foundations for advanced topics such as antiderivatives (integrals). To study the concept of derivatives, students must first have a solid understanding of algebra and the concept of function limits. The derivative of function is widely applied in real-life situations, such as determining the slope of a line, identifying the intervals where a function is increases or decreases, calculating the maximum profit of a company, minimizing losses in a project, instantaneous velocity, initial acceleration of an object, population growth rate, and many others. The numerous benefits gained from learning derivatives demonstrates the importance of thoroughly understanding this material for students. However, in practice, many students struggle to understand the concept of derivatives, causing a frequent errors in solving derivative application problems. This is based on an interview with a mathematics teacher at SMAN 6 Kota Cirebon, who revealed that one of the most challenging topics for students, with frequent errors in its execution, is solving problems related to the algebra of derivatives. This aligns with the research conducted by Fransiska et al. [6], which showed that the mathematical competence of eleventh-grade students at SMA Plus Al-Athiyah Tahfidz Al-Quran Banda Aceh in solving algebraic derivative problems was largely classified as low, with 54% of students in this category. Research on error analysis in student work was also conducted by Nurazizah et al. [7] on algebraic derivative problems. The results showed that students made calculation errors 2.5% of the time and conceptual errors 6.25% of the time. The causes of these errors were identified as a lack of deep understanding of the concept of derivatives, failure to recall formulas, and frequent mistakes in derivative operations. This indicates the need for a deeper analysis of students' errors, so that appropriate solutions can be found. Therefore, this study aims to analyze students' errors in solving derivative application problems based on Watson's criteria.

II. METHOD

This study employs a descriptive analysis method with a qualitative approach. The data analyzed in this study were derived from the formative assessment on the topic of derivatives of functions administered to students in Class XII MIPA 3 at SMAN 6 Cirebon. Among the 34 students who participated, 13 exhibited five distinct types of errors in their responses. These errors were analyzed based on the error criteria guidelines proposed by Watson, which include: (1) inaccurate data, (2) incorrect procedure, (3) missing data, (4) missing conclusion, (5) response level conflict, (6) indirect manipulation, (7) skill hierarchy issues, and (8) errors other than the seven listed [8]. The test instrument that used consisted two word problems related to the application of function derivatives, with the detailed distribution of the test items presented in Table 1.

TABLE I. SPECIFICATION OF TEST ITEM DISTRIBUTION

Basic Comptence	Indicator of Question	Question Number	Cognitive Aspect
Students are able to solve problems related to the application of function derivatives in everyday life.	Solve problems of trigonometric function derivatives involving the initial velocity of a wave.	1	C_3
	Determine the solution of function derivative applications involving maximum profit from a chip company.	2	C_3

The interview was conducted with three subjects to analyze the errors they made in solving problems related to the application of function derivatives. This study employed semi-structured interviews guided by an interview framework, the interview results were presented in the form of transcripts. The data was then presented as narrative text. In the final stage, the researcher drew conclusions based on the test results and interviews, which were analyzed in accordance with the research questions.

III. RESULTS AND DISCUSSION

Previous research analyzing students' errors has been conducted by Paiderowi et al. [9] on junior high school students in grades VII-IX from South Tangerang City and Ambon City. The study found that the errors students made in solving mathematical problems on the topic of ratio included conceptual errors, operational errors, principal errors, and other types of mistake. Similar errors also occurred when students attempted to solve problems related to function derivatives. According to the research by Istifaroh and Suparman [10], students made conceptual errors at a rate of 3.03% and computational errors at a rate of 20.3%. The contributing factors include a lack of understanding of the concept of derivatives, particularly in simple trigonometric derivatives and composite derivatives using the chain rule, as well as errors in basic numerical operations, such as multiplying functions, using parentheses, and performing operations involving negative signs. Therefore, the researcher aims to analyze the errors made by students in solving problems related to the application of derivatives, based on the criteria established by Watson. The instrument used in this study was a formative test. After analyzing the results of the test from 34 students, it was found that there were several errors that they made in solving the story problems. The analysis revealed that 13 students made errors in their answers. The derivative application problem given to 12th Grade MIPA 3’s students are presented below:

TABLE II. PROBLEM STORY

M1	M2
wave propagates with the equation $\boldsymbol{y = 3\cos\left(2t - \frac{\pi}{6}\right)}$, where y is in meters and ttt is in seconds. Determine the initial velocity of the wave!	A company produces x units of chips at a cost of $\boldsymbol{4x^2 - 8x + 24}$ thousand rupiahs per unit. If the chips are sold out at a price of 40 thousand rupiahs per unit, determine the maximum profit the company can achieve!

M1 represents the first problem, while M2 represents the second problem. After reviewing the test results, the researcher selected the students' answers to M2 for further analysis. The reason is that there were students who made errors in solving M2. In analyzing errors based on Watson's criteria, indicators are required as presented in the table below.

TABLE III. INDICATORS OF WATSON'S ERROR CRITERIA

Watson's Error Criteria	Indicators
Inappropriate Data (ID)	a. The formula used is incorrect
	b. Incorrect data input into the variable (incorrectly entering known data into the provided information)
Inappropriate Procedure (IP)	a. Not using the correct method in the process
	b. Writing steps that do not align with the problem
Missing Data (MD)	Incomplete data entry, losing one or more pieces of data
Missing Conclusion (MC)	Not using the obtained data to make a conclusion based on the problem's answer
Response Level Conflict (RLC)	Lack of understanding of the problem's intent
Undirect Manipulation (UM)	Using random or illogical reasons in the process
Skill Hierarchy Problem (SHP)	a. Unable to express algebraic ideas
	b. Making errors in calculations
Above Other (AO)	a. Rewriting the question
	b. Not providing an answers
	c. Writing a solution not according to the instructions

The following are the types of errors commonly made by students in solving M2:

- Missing Conclusion (MC): 2 students
- Skill Hierarchy Problem (SHP): 5 students
- Missing Data (MD): 4 students
- Inappropriate Procedure (IP): 1 student
- Response Level Conflict (RLC): 1 student

Additionally, six students exhibited a variety of errors in their test results when solving the problem, including:

1) Subject R made a Skill Hierarchy Problem (SHP) error by failing to simplify the quadratic equation

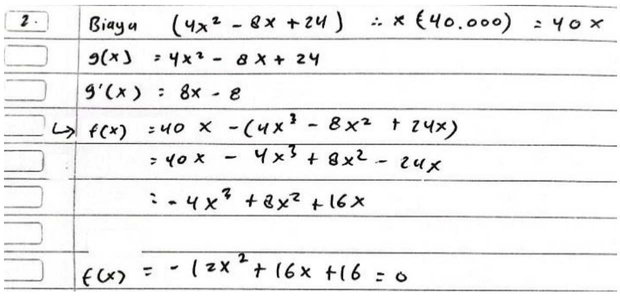

Figure 1. Subject R's Answer

Based on the answer above, it is evident that Subject R was unable to express algebraic ideas in solving the problem. They assumed that the cost or g(x) should also be derived, which is not the correct step. Here is the interview's result:

Researcher	:	*For question number 2, what information do you know from the problem?*
Subject R	:	*The cost or g(x) is $4x^2 - 8x + 24$, and the price per unit is 40,000.*
Researcher	:	*What is your first step in solving the problem?*
Subject R	:	*The first step I took was to make assumptions, namely the cost function g(x) and the profit function h(x)*
Researcher	:	*What is the next step?*
Subject R	:	*Since the selling price per unit is 40,000, I wrote 40x, Ma'am. The profit can be obtained by subtracting the initial cost from 40x.*
Researcher	:	*Why did you find the derivative of g(x)?*
Subject R	:	*I was confused, Ma'am. Usually, when there is an equation, I find its derivative.*

Subject R knows some terms in the problem story but is still confused about executing the solution.

Researcher	:	*What made you unable to solve it?*
Subject R	:	*I didn't really understand what was meant, and there were so many numbers I was confused about, including what they represented and how to apply them, Ma'am. So, it took me a long time to understand the problem.*
Researcher	:	*What obstacles did you face when solving the problem?*
Subject R	:	*I was pressed for time, so I didn't have a chance to factor the equation f(x), Ma'am.*

The results of the student's work and the interview indicate that Subject R is capable of expressing dialy events in mathematical language or symbols. However, errors occurred in solving the problem, particularly the inability to determine the roots of the quadratic equation they had written, due to running out of time. This aligns with the study conducted by Lubis, A.N. et al. [11], which found that 20 students from SMA Negeri 1 Pancur Batu were unable to solve derivative problems involving applications and analysis requiring deeper understanding. These included subtopics like determining increasing and decreasing functions as well as the maximum and minimum values of a function. The students were unable to answer the questions because they either did not understand these subtopics or required an extended amount of time to solve such derivative problems.

2) Subject D made an error categorized as a Skill Hierarchy Problem (SHP) during the calculation process.

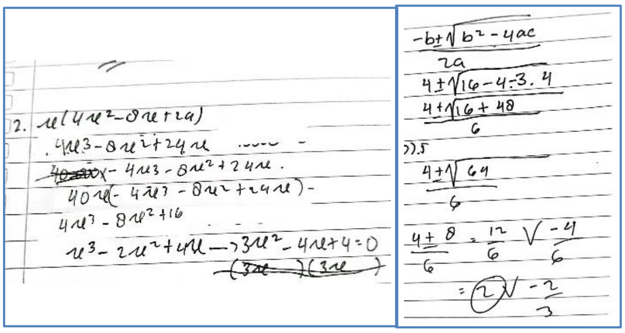

Figure 2. Subject D's Answer

Subject D made an error in the distributive process above. Misplacement of the negative sign led to a calculation mistake. The correct expression should be written as $40x - (4x^3 - 8x^2 + 24x)$ resulting in $-4x^3 + 8x^2 + 16x$. However Subject D wrote $4x^3 - 8x^2 + 16x$ which was then simplified incorrectly to $x^3 - 2x^2 + 4x$

In solving the problem, several errors were identified in Subject D's response. One notable mistake was a sign error, as the answer on the left-hand side ($a = 3$) differed from that on the right-hand side ($a = -3$). During the process of finding the roots of the quadratic equation, Subject D substituted the value of a as -3, which led to an inconsistency in notation and affected the final result.

This aligns with the findings of Hamid, et al [12], which highlight that students often struggle with consistency in mathematical notation in solving story problems related to a system of three-variable linear equations. In this context, students demonstrated a lack of accuracy in applying calculation skills, often making errors in mathematical operations. Some students struggled with performing addition, subtraction, multiplication, and division operations correctly, leading to incorrect results. Additionally, Subject D was unable to complete the problem because did not substitute x=2 into the original equation to determine the maximum profit. Below is the interview's result:

Researcher	:	*How did you solve this problem?*
Subject D	:	*I represented the number of units with x. Then, I multiplied x with the given equation. The price per unit was also multiplied by x, so it became 40x.*
Researcher	:	*What did you do next?*
Subject D	:	*I subtracted the equation from 40x, and I got the equation* $-4x^3 - 8x^2 + 16x$*. After that, I differentiated it and set it equal to 0. Then I tried to find the roots, ma'am*
Researcher	:	*So, from this problem, why couldn't you solve it?*
Subject D	:	*I got confused with factoring, ma'am. So, I used the ABC formula. I didn't have time to substitute x = 2 into the original equation. The bell rang too soon. So, I just left it as it was.*

Based on the results of the student's answer and the interview, overall, Subject D was able to understand the problem by correctly identifying what was being asked in the question. However, Subject D was in a hurry to solve the problem, so he was less careful and made a mistake in writing the operation signs. Subject D was unable to solve the problem because he was encountered during the process of finding the roots of the quadratic equation. In the calculation stage, Subject D used the quadratic formula to determine the roots of the quadratic equation. If subject D had substituted x = 2 into the original equation, he would have been able to solve the M2 problem.

3) Subject F made a Missing Data (MD) error due to incomplete data input and losing data during the problem-solving process for M2.

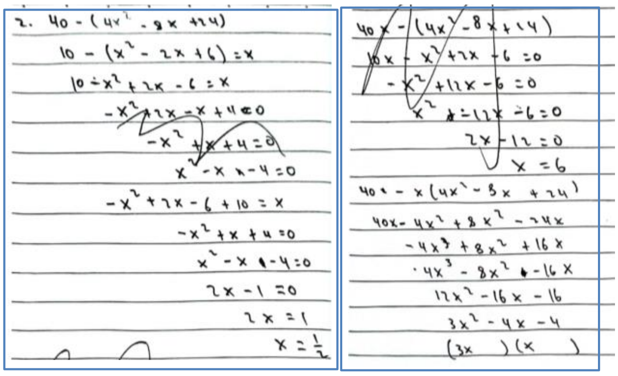

Figure 3. Subject F's Answer

The response above indicates that Subject F attempted to solve the problem using various methods, despite misconceptions in the initial steps. Below is an excerpt from the interview:

Researcher	:	*What information do you know from question number 2?*
Subject F	:	*The price per unit is (4x²-8x+24) thousand rupiah. It's sold out at x units for 40,000. The question asks for the maximum profit.*
Researcher	:	*Did the "thousand rupiah" detail make it difficult for you?*

Subject F : *Not really, Ma'am. I was more confused about where to place the 40,000 and where to put the price per unit.*

Researcher : *Which part do you find difficult, causing the solution to remain incomplete?*

Subject F : *It's because I was still confused during the factoring process, Ma'am—where to place this number and where to place the others.*

Subject F was unable to complete the factoring process, which resulted in the failure to obtain accurate data for finding the roots of the quadratic equation. This was the reason Subject F could not proceed to the next steps of the solution. This is consistent with the findings of the research by Usqo, U. et al. [13], which stated that the cause of missing data errors is due to a lack of understanding of the given problem and students forgetting how to solve it, resulting in their inability to continue the problem-solving process until the final stage.

4) Subject H made an Inappropriate Procedure (IP) error by not using the correct method in solving the M2 problem.

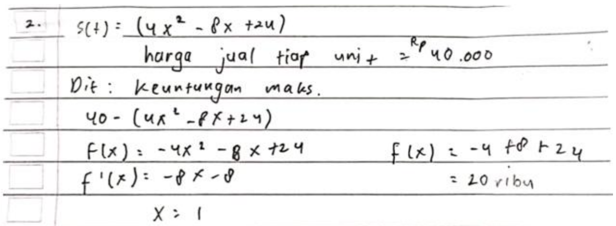

Figure 4. **Subject H's Answer**

Based on the above response, Subject H wrote steps that did not align with the problem. First, Subject H made an error when writing the production cost assumption as s(t), even though the problem already provided the equation with the variable x, so it should have been written as s(x). Second, Subject H did not understand the purpose of the problem, which led to the incorrect use of algebraic concepts. In the response, it was evident that Subject H did not associate the number of units of chips with the variable x, causing confusion when dealing with the statement about chips being sold at 40,000 per unit. Because Subject H did not multiply s(x) by x, the solution became inaccurate, leading to incorrect answers in subsequent steps (despite Subject H seemingly understanding the definition of the derivative concept). As a result, the direction of the solution became unclear and led to an Inappropriate Procedure (IP) error. Subject H declined to be interviewed due to embarrassment

5) Subject Z made a Response Level Conflict (RLC) error due to a lack of understanding of the problem and only writing a small portion of what was known about the issue.

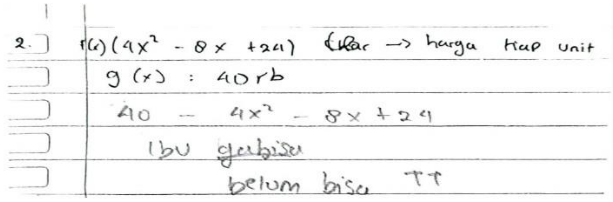

Figure 5. **Subject Z's Answer**

Subject Z made an error in formulating the mathematical model and did not know the steps to solve problem number 2. Subject Z declined to be interviewed. This female student could not solve the problem due to not understanding the solution process, leading to a Response Level Conflict (RLC) error. This aligns with the research conducted by Rusdiati et al. [14], which found that female students in class VIII at MTsN 5 Pesisir Selatan made errors categorized as Response Level Conflict (RLC) when solving problem story involving systems of linear equations with two variables, while male students did not make this type of error.

6) Subject M made a Missing Conclusion (MC) error by not using the data that had been obtained to draw a conclusion from the problem's solution.

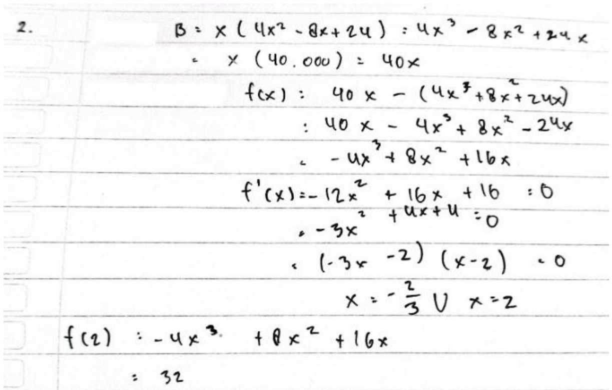

Figure 6. **Subject M's Answer**

Figure 6 shows that Subject M is able to understand the problem by accurately writing down the given information. Overall, Subject M almost completed the problem-solving process. However, the researcher observed that the answer provided by Subject M did not align with the problem's requirement, which was to determine the maximum profit in thousands of rupiah. The error made by Subject M was the failure to write a concluding statement for the answer obtained. This is consistent with the findings of Fahlevi, M. S., & Zanthy, L. S. [15], which state that the conclusion omission errors made by the students occur because the students fail to write a conclusion at the end of their answers, even though they have reached the final result of solving the problem. Additionally, these errors are also caused by the students not completing the problem-solving process. Subject M declined to be interviewed.

IV. CONCLUSIONS

Based on the data analysis from the conducted study, it can be concluded that several errors were identified among the students of Class XII MIPA 3 in solving problems related to the application of derivatives of functions. Five main types of errors were identified according to Watson's error criteria: inappropriate procedure made by subject H, skill hierarchy problem observed in subjects D and R, missing data error made by subject F, response level conflict error made by subject Z, and missing conclusion made by subject M. These errors were caused by various factors, including misconceptions in writing down known mathematical ideas, insufficient mastery of basic mathematical skills, negligence in calculations, particularly in algebraic operations involving finding the roots of quadratic equations, and lack of knowledge of the appropriate procedures for solving contextual problems related to the application of derivatives of functions.

It is important for mathematics educators in high school to pay attention to students' problem-solving abilities and develop their critical thinking skills. This will help students avoid similar mistakes in the future and enhance their problem-solving skills in mathematics, particularly in the application of derivative functions. The researcher recommends that an in-depth analysis of students' errors based on Watson's criteria be conducted, considering factors such as prior knowledge, learning styles, and motivation. This approach will lead to more effective mathematics learning, optimizing students' potential and creativity through methods tailored to their individual needs.

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