

## **Introducing Dynamic Mathematics Software to Secondary School Teachers: The Case of GeoGebra**

JUDITH HOHENWARTER AND MARKUS HOHENWARTER

*Florida State University*

*USA*

[jhohenwarter@lsi.fsu.edu](mailto:jhohenwarter@lsi.fsu.edu)

[mhohenwarter@lsi.fsu.edu](mailto:mhohenwarter@lsi.fsu.edu)

ZSOLT LAVICZA

*University of Cambridge*

*UK*

[zl221@cam.ac.uk](mailto:zl221@cam.ac.uk)

This article describes a study aimed to identify most common impediments related to the introduction of an open-source mathematical software package GeoGebra. We report on the analysis of data collected during a three-week professional development programme organised for middle and high school teachers in Florida. The study identified challenges participants face during workshops and evaluated the difficulty levels of GeoGebra tools. Findings of the study, complexity criteria of software tools and commonly occurring difficulties, provided the basis for the development of several new materials assisting workshop activities and contributed to the improvement of introductory GeoGebra workshops.

Technology is becoming an increasingly important factor in everyday life and computers are available virtually everywhere particularly in developed countries. At the same time, educational organisations have started to develop technology-related standards (Lawless & Pellegrino, 2007) trying to foster the integration of technology into teaching and learning. For example, the National Council of Teachers of Mathematics' (NCTM) *Principles and*

*Standards for School Mathematics* dedicated technology as one of their six principles for school mathematics: “*Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning.*” (NCTM, 2000, p. 11)

Students can benefit in different ways from technology integration into everyday teaching and learning. New learning opportunities are provided in technological environments, potentially engaging students of different mathematical skills and levels of understanding with mathematical tasks and activities (Hollebrands, 2007). In addition, the visualisation and exploration of mathematical objects and concepts in multimedia environments can foster understanding in new ways. Van Voorst (1999, pp. 2) highlighted that technology was “*useful in helping students view mathematics less passively, as a set of procedures, and more actively as reasoning, exploring, solving problems, generating new information, and asking new questions.*” Furthermore, he claimed that technology helps students to “*visualize certain math concepts better*” and that it adds “*a new dimension to the teaching of mathematics*” (p. 2).

Technological environments may also allow teachers to adapt their instruction and teaching methods more effectively to their students’ needs (NCTM, 2000). By integrating educational tools into their everyday teaching practice, they can provide creative opportunities for supporting students’ learning and fostering the acquisition of mathematical knowledge and skills. Additionally, on the one hand, gifted students can be supported effectively by nurturing their individual interests and mathematical skills. On the other hand, weaker students can be provided with activities that meet their special needs and assist them to overcome their individual difficulties. Therefore, students “*may focus more intently on computer tasks*” and “*may benefit from the constraints imposed by a computer environment*” (NCTM, 2000, p. 24). Furthermore, students can develop and demonstrate deeper understanding of mathematical concepts and are able to investigate more advanced mathematical contents than in “traditional” teaching environments.

Although the potential benefits of technology use for teaching and learning are well known and extensively examined, the process of integrating technology into mathematics classrooms proved to be slower than initially expected (Cuban, Kirkpatrick, & Peck, 2001). While many teachers are willing to experiment with new technologies they often are hindered by initial difficulties such as the lack of access to technology, basic skills for using the new technologies, and knowledge about effective integration of new tools into their teaching practices (Lawless & Pellegrino, 2007; Mously, Lambdin, & Koc, 2003; Swain & Pearson, 2002). In addition, many teachers

do not feel comfortable using computers or do not recognise how technology can assist them in providing engaging and meaningful learning environments for their students (Niederhauser & Stoddart, 1994).

The NCTM *Principles and Standards for School Mathematics* states that “*effective use of technology in the mathematics classroom depends on the teacher.*” (NCTM, 2000, p. 25). However, a number of impediments caused by teaching environments and classroom management – e.g., availability of computers and software, standards driven curricula, time constraints, lack of support from colleagues and school administrators – made it difficult for teachers to tackle the extensive and demanding task of technology use in classrooms. Furthermore, research also indicates that solely providing technology to teachers in the majority of cases is insufficient for a successful integration of technology into teaching practices (Cuban, Kirkpatrick, & Peck, 2001). Nevertheless, it has been suggested that adequate training and collegial support have the potential to boost teachers’ willingness to integrate technology into their teaching and to develop successful technology-assisted teaching practices (Becker, Ravitz, & Wong, 1999). In order to support teachers with the challenge of successfully integrating technology into teaching and learning of mathematics, professional development opportunities were created in order to foster change in teaching practice in the short run and cause improvement of student achievement in the long run (Lawless & Pellegrino, 2007).

## THE DYNAMIC MATHEMATICS SOFTWARE GEOGEBRA

The study described in this article aimed to identify effective approaches for introducing dynamic mathematics software to secondary school mathematics teachers in order to provide a basis for the development of corresponding instructional materials for teacher professional development (Preiner, 2008). For high quality professional development, “*it is important to know in which way a software package can be introduced to novices most effectively*” (Mously, Lambdin, & Koc, 2003, p. 401) in order to minimise unnecessary difficulties and impediments during the introduction process for teachers and to facilitate the first contact with the new software tool as much as possible. Teachers who feel comfortable with operating a new software tool are more likely to integrate this tool into their teaching practices than teachers who experienced initial difficulties (Mously et al., 2003). In this study, the open-source software GeoGebra was selected from the pool of available software packages for mathematics teaching and learning (e.g., dy-

dynamic geometry software (DGS), computer algebra systems (CAS), spreadsheets), because GeoGebra is a versatile tool that combines the ease of use of DGS with features of CAS.

GeoGebra is freely downloadable from the Internet and thus it is available both in schools and at home without any limitations (Hohenwarter & Lavicza, 2007). To date there is only limited research available in relation to the effective integration of GeoGebra into teaching and learning mathematics. However, research on other DGS packages suggests that dynamic software can be effectively integrated into mathematics education and have the potential to foster student-centred and active learning (Sträßer, 2001; Laborde, 2001; Erez & Yerushalmy, 2006; Sträßer, 2002; Jones, 1999). Beyond DGS features GeoGebra also offers additional algebraic and graphical representations of mathematical objects which can also contribute to students' deeper understanding of mathematical concepts (Duval, 1999). Developing dynamically connected DGS and CAS features within a single software package was one of the desires of researchers for development (Schumann, 1991; Schumann & Green, 2000). GeoGebra not only offers a novel dynamically connected learning environment, but also its development aimed at delivering a software package that can be utilized in a wide range of grade levels. By attempting to follow the 'KISS' principle ('keep it short and simple'), developers emphasised that users should be able to use the software intuitively without having advanced computer skills (Hohenwarter, 2006). Furthermore, GeoGebra features easily web-exportable, so called *dynamic worksheets* that "have led many teachers to foster experimental and discovery learning for their students and to share thousands of such worksheets on the GeoGebraWiki." (Hohenwarter & Lavicza, 2007, p. 51)

## DESCRIPTION OF THE STUDY

The study described in this article was carried out with the aim to identify difficulties and impediments that participants face during GeoGebra workshops and to assess the usability of the software itself (Preiner, 2008). Overall, there were three main objectives for this study, namely: 1) to assess GeoGebra's usability and to identify those challenging features and tools that could cause difficulties during the introduction of GeoGebra; 2) to establish complexity criteria for assessing and categorizing dynamic geometry tools and their difficulty levels to be able to better accommodate needs of novice users in future workshops; and 3) to provide a basis for the improvement of introductory GeoGebra materials and technology-enhanced professional development of secondary school teachers.

## CONTEXT AND ENVIRONMENT

The study was implemented during a teacher professional development summer institute of the *Math and Sciences Partnership* project between the Department of Mathematical Sciences at Florida Atlantic University and the School Board of Broward County funded by the U.S. National Science Foundation. The two-week institute involved workshops introducing a selection of software packages (e.g., GeoGebra, Excel) that the participating in-service middle and high school mathematics teachers may utilize in their teaching when returning to their schools. During the first week of the summer institute, 44 teachers participated in four daily 70-minute GeoGebra workshops. Participating secondary school teachers were divided into three groups and participated in identical workshops given by one of the authors of this article and were assisted by several experienced GeoGebra users. During the workshops, participants used their own laptops and wireless Internet connection was provided in the workshop venue. At the beginning of the summer institute participants installed GeoGebra on their computers and throughout the workshop they were able to access all course materials, particularly homework exercises, on a designated website. This setup enabled all participants to practice their learned skills daily.

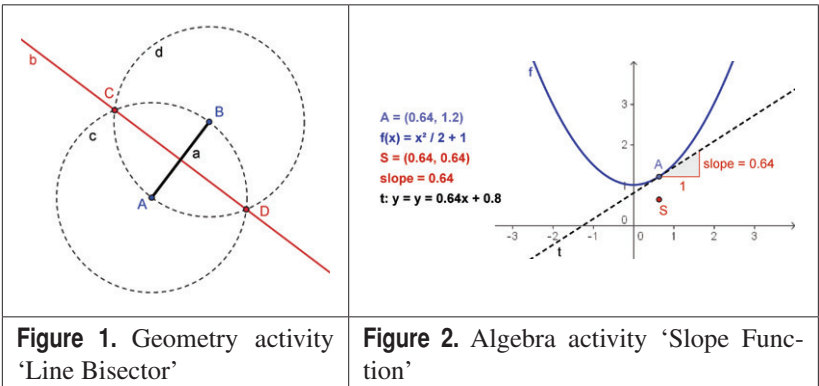
## DESIGN AND CONTENT OF INTRODUCTORY WORKSHOPS

The four workshops evaluated in this study were designed to introduce GeoGebra to novice users. Furthermore, workshops highlighted additional opportunities that GeoGebra can offer compared to other DGS packages. Hence, beyond geometry, workshops allowed the introduction of topics from algebra and calculus. Although workshops were tailored for secondary school teachers in Florida their design was based on experiences gathered from other workshops previously held in Europe and the U.S. As a routine, workshops began with discussing homework exercises followed by activities designed to enable participants' independency in GeoGebra use. After every workshop teachers were required to complete a short homework exercise.

The content of workshops was selected from well-known topics for Florida teachers. Topics ranged from geometric constructions, through experimenting with linear equations, to exploring functions in GeoGebra. For instance, one of the first geometry activities was the construction of the line bisector of a given segment. After practising the construction steps by creating a paper-and-pencil construction, participants were guided through the

GeoGebra construction (Figure 1). Consequently, participants were introduced to five GeoGebra tools and to the so called “drag test” allowing participants to dynamically modify constructions and explore their correctness.

By the end of the workshop series participants learned to model dynamic algebraic constructions (Figure 2). In this way they not only acquired knowledge about geometric manipulations but also engaged in using algebraic features of GeoGebra.



### EVALUATION INSTRUMENTS

The four GeoGebra introductory workshops were evaluated using a series of questionnaires<sup>1</sup> (Table 1). To ensure anonymity all questionnaires were labeled with an individual code.

**Table 1**  
Participant Surveys

Day	Survey	Filled in at	Content
1	Survey I	Beginning of WS I	Computer literacy
1	Workshop I	End of WS I	Activities and GeoGebra tools
1	Home Exercise I	At home	Exercise and GeoGebra tools
2	Workshop II	End of WS II	Activities and GeoGebra tools
2	Home Exercise II	At home	Exercise and GeoGebra tools
3	Workshop III	End of WS III	Activities and GeoGebra tools
3	Home Exercise III	At home	Exercise and GeoGebra tools
4	Workshop IV	End of WS IV	Activities and GeoGebra tools
4	Home Exercise IV	At home	Exercise and GeoGebra tools
5	Survey II	Beginning of day 5	GeoGebra features
10	Survey III	End of Institute	Mathematics content knowledge

Survey I aimed to evaluate participants' computer literacy and habits. Four questionnaires (End of WS I, II, III, and IV) were designed to obtain written feedback and ratings of the difficulty level of the workshop activities, the geometry tools introduced, as well as algebraic input and commands. In addition, participants filled in a questionnaire, similarly rating homework exercises and GeoGebra tools required to use for their assignments. Furthermore, participants offered feedback about the time required to complete assignments and their perceived progress. Survey II was implemented at the end of the workshop period and was similarly structured as the previously described questionnaires. In addition to the questionnaires, Workshop assistants completed "helper report cards" in which they recorded emerged difficulties during the sessions. Survey III was designed by the official evaluator of the summer institute and aimed to measure participants' mathematical content knowledge using 24 questions aligned with the Florida Sunshine State Standards.

## SUMMARY AND INTERPRETATION OF RESEARCH FINDINGS

In this section some results of the findings organized around the research questions posed by Preiner (2008) are highlighted.

### INTRODUCTORY WORKSHOPS

Question 1: *Were design, content, and difficulty level of the introductory workshops appropriate for secondary school teachers?* The study questionnaires utilised a uniform Likert scale ranging from 0 ('very easy') to 5 ('very difficult') to assess difficulty levels of both workshops and GeoGebra tools. Based on the low difficulty ratings for workshop activities (between 1.02 and 2.05) it can be conjectured that the workshops were feasible and appropriate for the participating secondary school teachers. In the written responses participants indicated that they overall enjoyed the workshops and benefited from its contents. Although the difficulty of three out of four homework assignments were rated as more complicated on average (2.19) than in-class activities (1.4), participants did not report specific difficulties using GeoGebra at home. In general, the data analysis revealed hands-on nature of sessions seemed to appeal most to participants and it helped to keep them focused and motivated throughout the institute.

Question 2: *How did teachers experience the introduction to GeoGebra and what kind of feedback did they give concerning its usability?* Overall, participants were most pleased with the usability and versatility of GeoGebra. They characterized GeoGebra as user friendly, easy and intuitive to use, and potentially helpful for teaching mathematics in secondary schools.

### **SUBJECTIVE DIFFICULTY RATINGS OF GEOGEBRA TOOLS**

Question 3: *Did users tend to subjectively rate GeoGebra's dynamic geometry tools to be of different difficulty levels when they were introduced in a workshop?* Although the average difficulty ratings of geometric tools initially was not high (1.8), differences in the complexity of tools had to be noted. However, after gaining more familiarity with GeoGebra participants rated all tools as "easy-to-use" (0.85 or less) regardless of their initial difficulty ratings.

Question 4: *Did the activities used to introduce a certain dynamic geometry tool have an impact on this tool's subjective difficulty rating?* Results suggested strong correlation (Spearman correlation coefficient=0.894) between difficulty ratings of tools and activities. Although it is difficult to establish the direction of causality, participants' and assistants' written feedback implied that complex tasks negatively influenced tool ratings. This claim certainly needs further examination, but future activity designs should consider simple exercises while introducing new tools and increase of the task complexity afterwards.

### **COMPLEXITY CLASSIFICATION OF DYNAMIC GEOMETRY TOOLS**

Question 5: *Was it possible to classify GeoGebra's dynamic geometry tools under groups of common characteristics that determine their general difficulty levels?* Difficulty ratings of tools identified common tool characteristics. Five complexity criteria for geometry tools emerged from the data and allowed the classification of all GeoGebra tools into three complexity-level groups (Preiner, 2008). This characterization provides an invaluable basis for the design of new materials for GeoGebra introductory workshops.



## GEOGEBRA FEATURES AND ALGEBRAIC INPUT

Question 6: *Did the use of specific GeoGebra features, algebraic input, or commands cause additional difficulties for participants?* Although there were only minor differences between difficulty ratings of geometric and algebraic tasks, participants tended to spend about 50% more time on tasks involving algebraic input or the use of commands. The written responses indicated difficulties resulting from learning inputting syntax, but this was anticipated as earlier studies highlighted the ease of use and intuitiveness of DGS as opposed to the steep learning curve required for CAS (Lavicza, 2008). Concerning GeoGebra's features, participants regarded features with more options (e.g., labels with options such as "name," "name and value," "value") to be more challenging to operate than those that can be simply turned on and off. In general, the way of accessing features in GeoGebra generally did not result in additional problems, although the right-click (MacOS: Apple-click) necessary to open the *Context menu* caused unexpected problems among Windows users.

## IMPACT OF EXTERNAL VARIABLES AND FREQUENTLY OCCURRING PROBLEMS

Question 7: *Did external variables such as math content knowledge, computer literacy, or the use of a touchpad, influence the subjective difficulty rating of GeoGebra, its tools, or its features?* External variables such as gender, age, teaching experience, mathematics content knowledge, computer skills, or different operating systems did not result in significant differences in any difficulty ratings. The only variable that influenced ratings significantly was the indicator of use of touchpad or computer mouse. Touchpad users rated workshop activities (1.84 versus 1.01), DGS tools (1.40 versus 0.62), and GeoGebra features (1.54 versus 0.70) more difficult than mice users. However, learning from this experience, workshop presenters are now encouraged to bring extra mice for the participants.

Question 8: *Which difficulties, problems, and questions occurred most often during the introductory workshops?* Written responses of participants and workshop assistants were analysed by utilizing constant comparative coding based on the Grounded Theory approach. It emerged from the data analysis that participants required assistance in constructing geometric figures, properly using dynamic geometry tools, and inputting appropriate algebraic

syntax. These difficulties were collected and provide a basis for the development of detailed guidelines (e.g., introductory book, workshop handouts) for future workshops.

## CONCLUSIONS AND FURTHER RESEARCH

In accordance with the literature, the successful integration of technology into mathematics teaching and learning is a rather complex and tedious process (Laborde, 2001; Lagrange et al., 2003). Access to technology is becoming increasingly widespread in both schools and at home and the quality of mathematical software packages is improving rapidly; nevertheless, technology is still marginally integrated into education at all levels (Lavicza, 2008). Studies highlighted that offering high-quality professional development for teachers is essential for successful technology integration. In this article, we highlighted the evaluation of a professional development programme with a versatile mathematical software package GeoGebra. Results of this study identified difficulties that teachers face while participating in technology-enhanced workshops and learning the use of new software applications. These findings allowed for the development of complexity criteria and classification of dynamic geometry software tools and pinpointed how technology professional development for teachers can be improved in the future. The study immediately resulted in improvement of consecutive series of workshops and the design of several new handouts and books to ease difficulties of novice users.

Based on these results the authors carried out follow-up projects to assess teachers' actual use of technology after returning to their schools. In addition, they improved the evaluation tools and continue to strive for improving professional development of teachers as well as developing appropriate materials. Extended results of this and the follow up study will appear in print in the near future.

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## Websites

GeoGebra home page: <http://www.geogebra.org>  
GeoGebraWiki: <http://www.geogebra.org/wiki>  
GeoGebra User Forum: <http://www.geogebra.org/forum>

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## Note

<sup>1</sup>A detailed description of the all evaluation instruments can be found in Preiner (2008).